## **Mathematical Modeling and Analysis**



# Light Propagation through Biological Tissue

Travis Austin, taustin@lanl.gov

#### Introduction

The propagation of light through human tissue is best described by the linear transport equation. This equation describes neutral particles (e.g., photons or neutrons) streaming through a physical medium. One example of such a medium would be water. Within the medium, the particles either stream uninterrupted or collide with the nuclei that constitute the medium. A collision results in either an absorption of the particle into the nucleus or a scattering to a new direction and new energy level. The application from biomedical optical imaging that is of interest is using this equation along side non-invasive techniques (like MRIs) for the detection of tissue abnormalities.

In my work, we define the solution to the transport equation as the discrete function that minimizes a pre-defined quadratic functional [1]. This principle provides us with a guide to use in developing a solution procedure, because it defines how to iteratively improve a discrete approximation to the true solution. Any improvement can be defined so that it guarantees a reduction in the error in an energy-like norm. This same principle from calculus allows one to find the minimum (or maximum) of a quadratic function.

### Recent Advances

Several recent changes have been made to the serial Fortran program that algorithmically represents the solution technique described above. These changes allow for the modeling of a broader range of problems. The initial program was only written for three-dimensional simulations. Recently, I adapted the code to allow for two-dimensional simulations. The problem with three-dimensions is that the corresponding serial Fortran code is extremely demanding of computer memory and also requires many computer operations to find an accurate solution. As a matter

of fact, we can not model realistic problems that require high accuracy without a parallel implementation of such a code. By investigating two-dimensional calculations, we can examine problems that need high accuracy. Hence, we can check the code's performance for a larger class of problems. Eventually, we can use information obtained from the two-dimensional simulations to improve the code for three-dimensions.

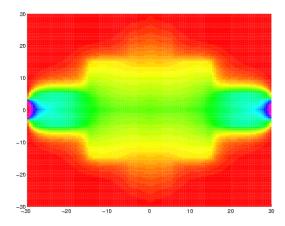


Figure 1. Scalar flux for a two-material steady-state problem with a boundary source at  $x = \pm 30$  and y = 0.

Two other features have also been added. One is the addition of reflective boundary conditions and the other is the addition of anisotropic scattering [2]. Reflective boundary conditions allow us to model problems that have an innate symmetry. This can be observed in Figure 1. We were able to solve this problem on the domain  $[0,30] \times [0,30]$ , instead of the full domain. Anisotropic scattering is necessary because it corresponds to what is seen with light propagation in biological tissue. With anisotropic scattering, we can account for particles that, after a collision with a nucleus, scatter in a preferential direction.

These new advances have allowed for the generation of the image seen in Figure 1. This image corresponds to the scalar flux, which is proportional to particle density, an example in which there is a source of photons on the boundary and two materials on the interior. The regions encompassing the two materials can be made out

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from the figure. One material is nearly a void. Here, particles have nothing to interact with. The second material is considered "thick"; thus, most particles entering this domain are scattered or absorbed by nuclei.

## Summary

These recent advances are part of a plan to eventually allow for adaptive mesh refinement and more meaningful physical problems. The long term goal is to use this code within impedance tomography problems, and thus use this for the detection of tissue abnormalities. Currently, this is often accomplished through the diffusion equation. The transport equation is a better model and should provide more accurate results.

## Acknowledgements

Los Alamos Report LA-UR-03-9065. Funded by the Department of Energy under contracts W-7405-ENG-36 and the DOE Office of Science Advanced Computing Research (ASCR) program in Applied Mathematical Sciences.

## References

- [1] T.A. Manteuffel, K. Ressel, and G. Starke, *A boundary functional for the least-squares finite element solution of the neutron transport equation*, SIAM Journal of Numerical Analysis, 35, pp. 806-835.
- [2] T.M. Austin and T.A. Manteuffel, A least-squares finite element method for the linear Boltzmann equation with anisotropic scattering, in preparation.